# Recitation, Week 6

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POL-850

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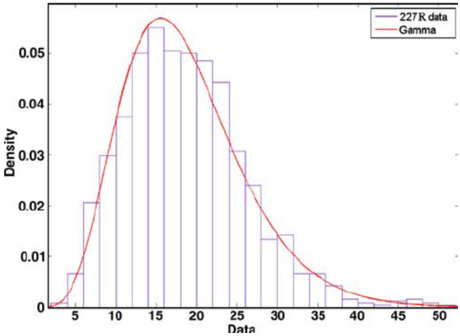
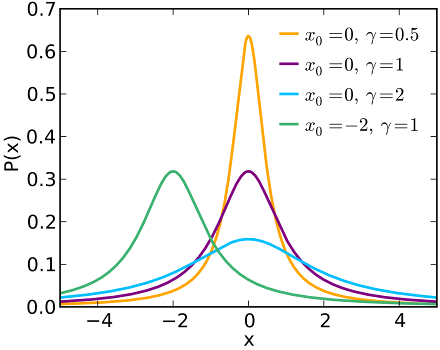
# Outline

# Probability Distribution




- Probability Distribution can be seen as the limit of a histogram
- X-axis: the index of events; Y-axis: the probability for the event to happen
- Imagine that we can measure X as precisely as possible...

# Probability Distribution




Properties of Probability Distribution

1. Y-value of the curve is always greater than or equal to 0
   $y = f(x) \geq 0$
2. The area underneath the curve is always 1
   $\int f(x)dx = 1$ (what does this integral mean?)

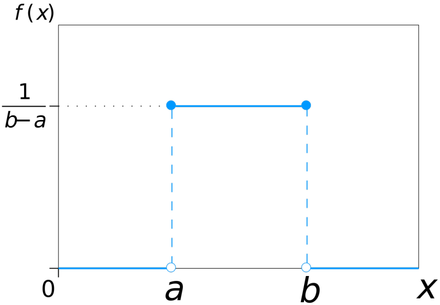
# Uniform Distribution



- Suppose you're waiting for a bus and you know that the bus comes every 5 minutes (but you don't know when the last bus passed the bust stop)
- X: the waiting time for the next bus
- X obeys uniform distribution ($X \sim U[0, 5]$)
- From now on, the probability for the bus to arrive is the same in any given time interval (e.g. one minute)

# Group exercise 1

- What is the probability that a bus comes between 0 and 5 minutes?
- Probability that a bus comes after 3 minutes?
- Probability that a bus comes between 2 minutes and 4 minutes?
- Why the value of the curve below takes always should take $\frac{1}{b-a}$?



# Standard Normal Distribution



We learned normal and standard normal distribution in class. But,

1. Why do we care about a normal distribution?
2. Why do we standardize using Z-score? (i.e. Why do we use the standard normal dist.?)

$$Z = \frac{X-\mu}{\sigma}$$

3. Why do we have the Z-table?

Because..

1. It is the most common distribution in real world and also in "statistical world" (due to the central limit theorem)
2. Normal distributions may have different means and standard deviation. By standardizing, we can convert all them into one distribution.
3. It helps us to calculate the area underneath the curve (i.e. to calculate a probability).

Simply put, we want to calculate a probability!

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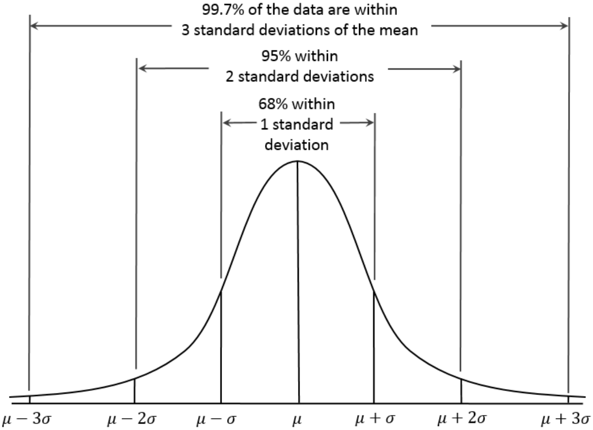

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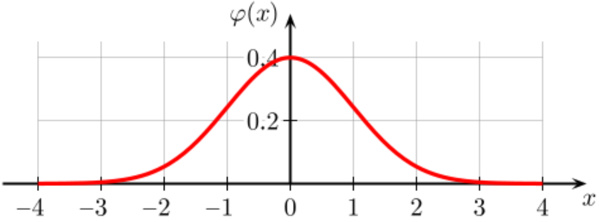

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$$Z = \frac{X - \mu}{\sigma}$$

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$$Z = \frac{X - \mu}{\sigma} = \frac{210 - 190}{20} = 1$$
- Looking at the Z-table returns $Pr(Z \geq 1) = 0.16$

# Group exercise 2

1. Visualize the previous result ($Pr(Z \geq 1) = 0.16$) using the graph of Standard Normal Distribution.
2. How to calculate the probability that a man randomly drawn from the population weighs greater than 180 pounds?
3. How to calculate the probability that a man randomly drawn from the population weighs greater than 170 and smaller than 200 pounds?

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- It's about finding a population parameter (population mean $\mu$)
- How do we get it without census?
- We randomly draw a sample from population
- And then we make a guess about the population parameter based on the sample

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- To understand the origin of confidence interval, we need to make sense of sampling distribution and central limit theorem
- That's because they form the basis of confidence interval

# Animation

A cool animation about rabbits' weight and dragons' wing span
(Length: about 4 minutes)

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- Therefore we can plot the sampling distribution of sample mean by:
- Randomly draw a sample $\rightarrow$ Calculate the sample mean $\rightarrow$ Repeat this process many times $\rightarrow$ Plot its distribution (or the histogram)

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- It implies that you don't need to care about the population distribution. You just form a sampling distribution with large sample size!
- That is enough for guessing a population mean (by calculating a confidence interval)